University of Saskatchewan

Department of Chemical Engineering
ChE 311– Mathematical Modelling I

Quiz #3

DATE:	**Wednesday December 1, 2004**
INSTRUCTOR:	**Professor T. Pugsley**
TIME:	**10:30 - 11:20 a.m., RM 1C70 Eng.**

Instructions: This is a closed book/closed notes quiz. Personal calculators are permitted. Write your answers neatly in the examination booklets provided. Please do both questions.

Question #1 (10 marks)

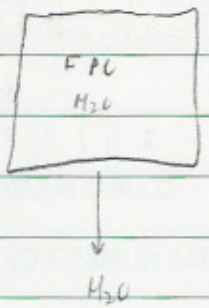
A plant at Canso, Nova Scotia, makes fish-protein concentrate (FPC). One of the operating problems is the drying of the FPC. It dries in the fluidized dryer rate a rate proportional to its moisture content. If a given batch of FPC loses one-half of its initial moisture in the first 15 min, how long will it take to remove 90% of the water in the batch of FPC?

Question #2 (10 marks)

A sewage disposal plant has a big concrete holding tank of 100, 000 gal capacity. It is ¾-full of liquid to start with and contains 60, 000 lb of organic material in suspension. Water runs into the holding tank at a rate of 20, 000 gal/h and the solution leaves at the rate of 15, 000 gal/h. How much organic material is in the tank at the end of three hours?

END OF EXAMINATION

① Given



→ 50% H_2O is lost after 15 min

Find → how long it will take to lose 90% H_2O

Solution

→ H_2O balance

Input : 0

Output : $kX\ \Delta t$

Acc : $(X|_{first} - (X)_t$

Acc : Input − Output

$\frac{dX}{dt} = 0 - kX$

$\frac{dX}{dt} = -kX$

$\int_{X_0}^{X} \frac{dx}{x} = -k \int_{0}^{t} dt$

$\ln \frac{X}{X_0} = -kt$

$\frac{X}{X_0} = e^{-kt}$

$X = X_0\, e^{-kt}$

→ known conditions

→ $t = 15$, $x = \frac{1}{2} x_0$

$x = x_0 e^{-kt}$

$\frac{1}{2} x_0 = x_0 e^{-15k}$

$\frac{1}{2} = e^{-15k}$

$-0.693 = -15k$

$k = 0.0462$ min^{-1}

→ time for $x = 0.1 x_0$

$x = x_0 e^{-kt}$

$0.1 x_0 = x_0 e^{-0.0462t}$

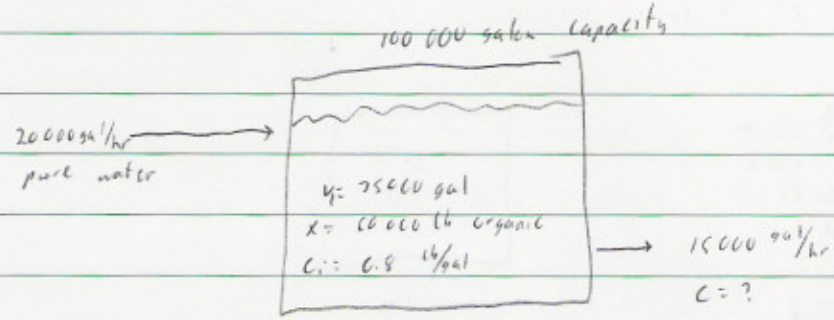
$0.1 = e^{-0.0462t}$

$-2.30 = -0.0462t$

$t = 49.78$

$t = \boxed{49.8 \text{ min}}$

② given



Find → how much organic material is in the tank at the end of 3 hours

Solution

→ H_2O balance

$Input = 20000\ \Delta t$

$Output = 15000\ \Delta t$

$Acc = (V)_{final} - (V)_i$

$Acc = Input - Output$

$\frac{dV}{dt} = 20000 - 15000$

$\frac{dV}{dt} = 5000$ ✓

$\int_{V_i}^{V} dv = 5000 \int_0^t dt$

$V - V_i = 5000\ t$

$V = 5000\ t + 75000$ ✓

→ at time $t = 3$ hr.

$V = 5000\ (3) + 75000$

$V = 90000$ gal ✓

→ organic balance

Input = 0

output = 15000 C Δt

Acc = $(VC)_{t+\Delta t} - (VC)_t$ ✓

Acc = Input − output ✓

$\frac{d(VC)}{dt} = 0 - 15000C$

$C\frac{dV}{dt} + V\frac{dC}{dt} = -15000C$ ✓

$\frac{dV}{dt} + \frac{V}{C}\frac{dC}{dt} = -15000$

$5000 + \frac{V}{C}\frac{dC}{dt} = -15000$

$\frac{V}{C}\frac{dC}{dt} = -20000$

$\int_{C_i}^{C} \frac{dC}{C} = -20000 \int_0^t dt$

$V \ln\left(\frac{C}{C_i}\right) = -20000\, t$

but this is not constant

$V = 5000t + 75000$

→ at time $t = 3$ hr

$75000 \ln\left(\frac{C}{0.8}\right) = -20000(3)$

$\ln\left(\frac{C}{0.8}\right) = -0.8$

$\left(\frac{C}{0.8}\right) = e^{-0.8}$

$C = 0.8\, e^{-0.8}$

$C = 0.8(0.449)$

$C = 0.359$ lb/gal ✗

$x = CV$

$x = (0.359 \text{ lb/gal})(90000 \text{ gal})$

$x = 32310$ lb

$x = \boxed{32306 \text{ lb}}$ ✗